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SEISMIC SLOPE STABILITY ANALYSIS: PSEUDO-STATIC GENERALIZED METHOD

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ABSTRACT

This paper extends a generalized slope stability analysis method to include pseudo-static forces. Formulation and the subsequent numerical procedure of the extended generalized seismic slope stability analysis are presented. Comparison with a closed form approach indicates that the generalized method yields the same safety factor as the closed form approach. This can serve as a partial verification of the accuracy of the numerical procedures. Comparison with other rigorous limit equilibrium methods of seismic slope stability demonstrates that the presented method yields the smallest factor of safety.

Key words: earthquake, safety factor, stability analysis, slope stability, slip surface, stress distribution (**IGC:** E6)

INTRODUCTION

An extension of Baker and Garbers variational limiting equilibrium approach to seismic slope stability analysis was proposed by Leshchinsky and San (1993). The variational approach yields a log-spiral slip surface for homogeneous problems. Complicated geological conditions, however, may require consideration of slip surfaces of a general shape. Generalized slope stability approach (Leshchinsky, 1990), which is partially based on variational analysis, can then be used. It satisfies all limiting equilibrium equations for a given slip surface.

This paper presents the formulation for the generalized approach to include seismic effects. A numerical scheme for computer programming of the generalized approach is proposed. Comparison between the closed form variational approach (Leshchinsky and San, 1993) and the generalized approach is conducted. Comparison with other rigorous methods of seismic slope stability analysis is also presented.

FORMULATION

In seismic slope stability design that is based on a limit equilibrium analysis, inertia forces are usually taken as pseudo-static, expressed as a fraction of the gravitational forces as defined by a design horizontal acceleration factor C_s; i.e., C_s is a fraction of the acceleration g (e.g., Sarma and Barbosa, 1985). The following is a brief presentation of analysis and results. Details and in-depth understanding of the analysis can be obtained with the aid of the provided references.

The potentially sliding mass is bounded by the soil surface and a slip surface, denoted by $\bar{y}=\bar{y}(x)$ and $y=y(x)$, respectively. The slip surface is acted upon by an unknown distributed normal stress $\sigma(x)$. Utilizing Coulomb's failure criterion and by a straightforward extension of Leshchinsky's (1990) formulation to include C_s, the pseudo-static limiting equilibrium equations for a sliding mass can be expressed as:

$$H=\sum_{j=m}^{1}\int_{x_j}^{x_{j-1}}\{c_j+(\sigma-u)\psi_j-F\sigma y' - C_s\gamma F(\bar{y}-y)\}dx=0 \quad (1)$$

$$V=\sum_{j=m}^{1}\int_{x_j}^{x_{j-1}}\{[c_j+(\sigma-u)\psi_j]y' - F[\gamma(\bar{y}-y)-\sigma]\}dx=0 \quad (2)$$

$$M=\sum_{j=m}^{1}\int_{x_j}^{x_{j-1}}\left\{[c_j+(\sigma-u)\psi_j](y-xy') - F[\sigma(yy'+x)-\gamma(\bar{y}-y)x] - \frac{1}{2}C_sF\gamma(\bar{y}-y)(\bar{y}+y)\right\}dx=0 \quad (3)$$

where H, V and M=the respective limiting equilibrium

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equations for horizontal forces, vertical forces and moments about the origin of the coordinate system; j=soil layer number (there are m layers through which the slip surface is passing); $y'=dy/dx$; $\psi_j=\tan(\phi_j)$, where ϕ_j is the internal angle of friction of layer j; c_j=cohesion of layer j; x_0 and x_n=the ordinates at which the slip surface intersects the slope surface (see Fig. 1), and x_j=the ordinate of the intersection with the lower boundary of layer j; γ=the weighed average unit weight of soil column $(\bar{y}-y)$; u=the pore-water pressure and F=a safety factor.

Using H to define F, and V and M as constraints, Baker and Garber (1978) showed the isoperimetic problem to be equivalent to the minimization of an auxiliary functional G. Including the seismic loading, G is defined as

$$G=\int_{x_0}^{x_n} g\,dx \tag{4}$$

where:

$$\begin{aligned} g=&\{c_j+(\sigma-u)\psi_j-F\sigma y'\} \\ &+\lambda_1\{[c_j+(\sigma-u)\psi_j]y'-F[(\bar{y}-y)-\sigma]\} \\ &+\lambda_2\{[c_j+(\sigma-u)\psi_j](y-xy') \\ &-F[\sigma(yy'+x)-\gamma(\bar{y}-y)x]\} \\ &-C_s\gamma F(\bar{y}-y)-\lambda_2 C_s\gamma F(\bar{y}-y)\frac{(\bar{y}+y)}{2} \end{aligned} \tag{5}$$

and λ_1, λ_2 are Lagrange's undetermined multiplies. Baker and Garber (1978) introduced the parameters x_c and y_c as a substitute to Lagrange's multiplies:

$$x_c=\frac{\lambda_1}{\lambda_2} \tag{6}$$

$$y_c=-\frac{1}{\lambda_2} \tag{7}$$

The unknown functions $y(x)$ and $\sigma(x)$ that minimize the functional G and produce $F_s=\min(F)$ should satisfy Euler's equations:

$$\frac{d}{dx}\frac{\partial g}{\partial y'}-\frac{\partial g}{\partial y}=0 \tag{8}$$

$$\frac{d}{dx}\frac{\partial g}{\partial \sigma'}-\frac{\partial g}{\partial \sigma}=0 \tag{9}$$

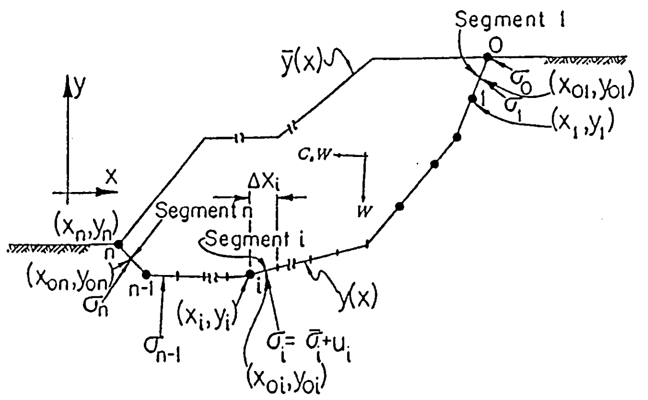

Fig. 1. Notation in numerical procedure

Combining the first Euler's equation with Eqs. (5), (6) and (7), and rearranging the terms give:

$$\begin{aligned} &[(x-x_c)\psi_j+(y-y_c)F_s]\sigma'+2\psi_j(\sigma-u)+2c_j \\ &-(x-x_c)(\psi_j u'+\gamma F_s)+C_s\gamma F_s(y_c-y)=0 \end{aligned} \tag{10}$$

where $\sigma'=d\sigma/dx$; $u'=du/dx$; (x_c, y_c)=two unknown geometrical constants; and $F_s=\min(F)$=the factor of safety. This differential equation contains a term related to C_s, a modification to Leshchinsky's (1990) solution.

The second Euler's equation should yield the critical slip surface $y(x)$. This $y(x)$, however, is limited to log spiral surfaces which may not always be realistic for layered slopes. In the generalized approach, an arbitrary $y(x)$, which can adapt to the local geology, is specified by the user. Then, the numerical solution of Eq. (10) gives $\sigma(x)$ containing three unknown constants F_s, x_c, y_c. Substituting $\sigma(x)$ in Eqs. (1), (2) and (3), and replacing F_s for F, one gets three nonlinear equations with three unknowns: F_s, x_c, y_c. Solving these equations yield F_s for the selected $y(x)$. Examining many potential slip surfaces $y(x)$ and calculating their respective F_s, the surface yielding the minimum F_s should be obtained. This absolute minimal F_s and its associated $y(x)$ are considered the critical results fulfilling the objective of the limiting equilibrium analysis.

NUMERICAL PROCEDURE

The numerical procedure follows the scheme presented by Leshchinsky and Huang (1992). Figure 1 shows the notation used in the procedure for the seismic slope stability analysis. First, the selected slip surface is discretized into n straight segments (i.e., 'slices'). Then, Eqs. (1), (2) and (3) can be rewritten in an approximated fashion as:

$$H=\sum_{j=1}^{m}\left\{\sum_{i=1}^{n}\delta[c_j+(\sigma_i-u_i)\psi_j-F_s\sigma_i y_i' - C_s\gamma_i F_s(\bar{y}_i-y_{0i})]\Delta x_i\right\}=0 \tag{11}$$

$$V=\sum_{j=1}^{m}\left(\sum_{i=1}^{n}\delta\{[c_j+(\sigma_i-u_i)\psi_j]y_i' - F_s[\gamma_i(\bar{y}_i-y_{0i})-\sigma_i]\}\Delta x_i\right)=0 \tag{12}$$

$$\begin{aligned} M=\sum_{j=1}^{m}\Bigg(\sum_{i=1}^{n}\delta\{&[c_j+(\sigma_i-u_i)\psi_j][y_{0i}-x_{0i}y_i'] \\ &-F_s[\sigma(y_{0i}y_i'+x_{0i})-\gamma_i(\bar{y}_i-y_{0i})x_{0i}] \\ &-\frac{1}{2}C_sF_s\gamma_i(\bar{y}_i-y_{0i})(\bar{y}_i+y_{0i})\}\Delta x_i\Bigg)=0 \end{aligned} \tag{13}$$

where i=slice number; x_{0i}, y_{0i}=coordinates of the center of the base of slice i; γ_i=weighted average total unit weight of slice i; $\bar{y}_i$=slope elevation avobe x_{0i}; $\Delta x_i = x_i - x_{i-1}$ and $y'=(x_i-x_{i-1})/\Delta x_i$.

The differential equation describing the total stress distribution, Eq. (10), can be rewritten for each slice as:

$$[(x_{0i}-x_c)\psi_j+(y_{0i}-y_c)F_s]\sigma_i'+2\psi_j(\sigma_i-u_i)+2c_j$$

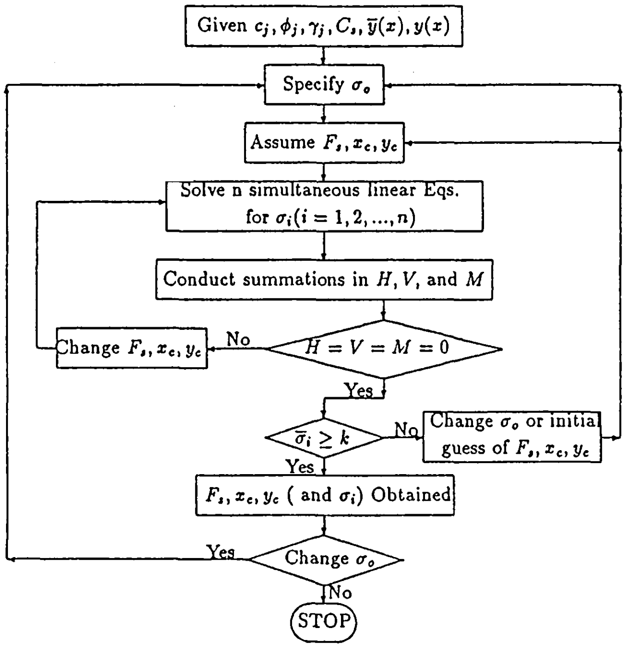

Fig. 2. Computation scheme

$$-(x_{0i}-x_c)(\psi_j u_i' + y_i F_s) + C_s \gamma_i F_s (y_c - y_{0i}) = 0$$

for $i = 1, 2, \cdots n$ (14)

The computation scheme for solving the problem is presented in Fig. 2. Notice that by using this scheme, the problem is reduced to three non-linear simultaneous equations and n simultaneous linear equations.

For given values of c_j, ϕ_j, C_s, $\bar{y}(x)$, and $y(x)$, with an initial specified value of σ_0 (see Fig. 1), and an initial guess of F_s, x_c, y_c, the proposed computation scheme is straightforward following these main steps:

Step 1: Solve n simultaneous linear equation [Eq. (14)] for $\sigma_i (i=1, 2, \cdots n)$;

Step 2: Change F_s, x_c and y_c until $H = V = M \cong 0$; for each change repeat Step 1. This is done automatically by a routine that solves simultaneous nonlinear equations;

Step 3: Change either σ_0 or the initial guess of F_s, x_c and y_c until $\bar{\sigma}_i (= \sigma_i - u_i)$ is not less than $k[= -(c/\tan \phi_j)]$; i.e., verify that for the obtained roots (F_s, x_c, y_c), the normal stress distribution does not violate Coulomb's strength by the inclusion of negative stresses in excess of admissible values.

Step 4: For the selected slip surface repeat steps 1, 2 and 3, by changing σ_0, until the lowest F_s and admissible σ_i are obtained.

COMPARATIVE RESULTS

The presented comparative study is limited to a two-part study. The first study is to compare the results obtained from the closed-form solution obtained by Leshchinsky and San (1993) with the generalized (i.e., numerical) approach. This provides some verification of the accuracy of the formulation of the proposed generalized approach. The second study compares the generalized approach with other methods. This provides a sense of whether the proposed method, which statically is assumption-free, yields a smaller factor of safety (i.e., "better" minimum as compared to other rigorous limit equilibrium methods where statical assumptions are utilized).

Two slope inclinations are considered in the first study: a vertical slope and a 1(V):2(H) slope. For the vertical slope, $\phi_m = 0$ and 35° with $C_s = 0.25$; for the slope inclined at 1:2, $\phi_m = 0$ and 15° with $C_s = 0.10$. A total of four cases are investigated, as summarized in Table 1. Note that $\phi_m = \tan^{-1} [(\tan \phi)/F_s]$ is the design (or mobilized) internal angle of friction. By utilizing the same critical slip surface obtained from the closed form variational solution, the generalized method yields the same value of Fs—see Tables 2 and 3. This is despite significant differences in the values obtained for X_c and Y_c. Since X_c and Y_c represent the pole of the log spiral, it is likely that more accurate representation of the smooth critical slip surface, $Y(X)$, in the input data would have resulted with higher accuracy of X_c and Y_c in the generalized approach. It is apparent, however, that F_s is insensitive with respect to X_c and Y_c. Since the objective is to find F_s, inaccuracies in X_c and Y_c are of lesser concern. Figures 3 to 6 show the comparison of the distribution of normal stress acting over the critical slip surface for all four cases. In these figures, a nondimensional notation, such as $X = x/H$, $Y = y/H$ and $S = \sigma/\gamma H$, is used. H in this nondimensional notation is the height of the slope. The agreement of the stress distribution obtained numerically with the

Table 1. Data for investigated cases

Case Number	Slope inclination	ϕ_m	$N_m = \frac{c}{F_s} \frac{1}{\gamma H}$	C_s
Case 1	Vertical slope	0.0	0.331	0.25
Case 2	Vertical slope	30.0°	0.218	0.25
Case 3	1(V):2(H)	0.0	0.203	0.10
Case 4	1(V):2(H)	15.0°	0.064	0.10

Table 2. Results obtained from variational closed form solution

Case Number	$X_c = \frac{x_c}{H}$	$Y_c = \frac{y_c}{H}$	F_s
Case 1	−1.598	3.298	1.000
Case 2	−1.863	5.995	1.000
Case 3	1.922	2.322	1.000
Case 4	1.175	2.310	1.000

Table 3. Results obtained from the generalized approach

Case Number	$X_c = \frac{x_c}{H}$	$Y_c = \frac{y_c}{H}$	F_s
Case 1	−3.358	3.592	1.001
Case 2	−1.650	2.531	1.001
Case 3	1.312	2.496	0.999
Case 4	0.566	2.940	0.999

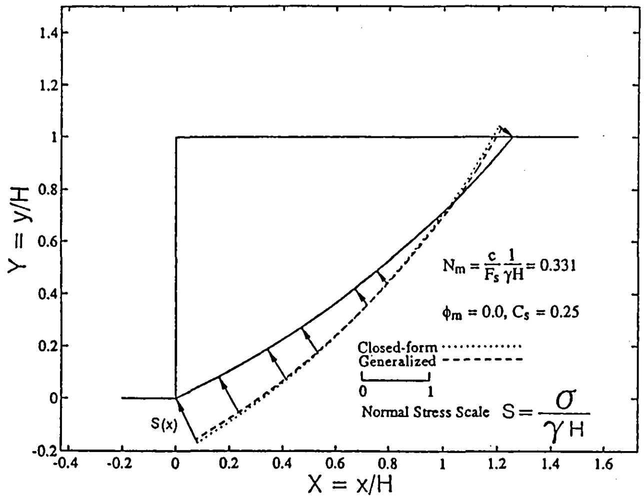

Fig. 3. Comparison between the closed-form approach and the generalized method: normal stress distribution (Case 1)

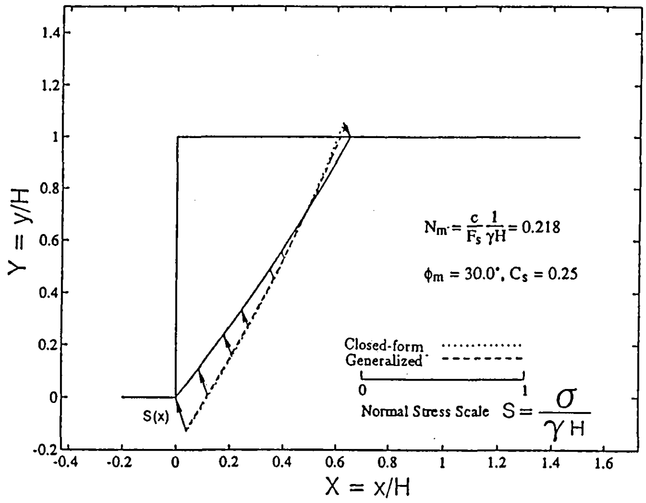

Fig. 4. Comparison between the closed-form approach and the generalized method: normal stress distribution (Case 2)

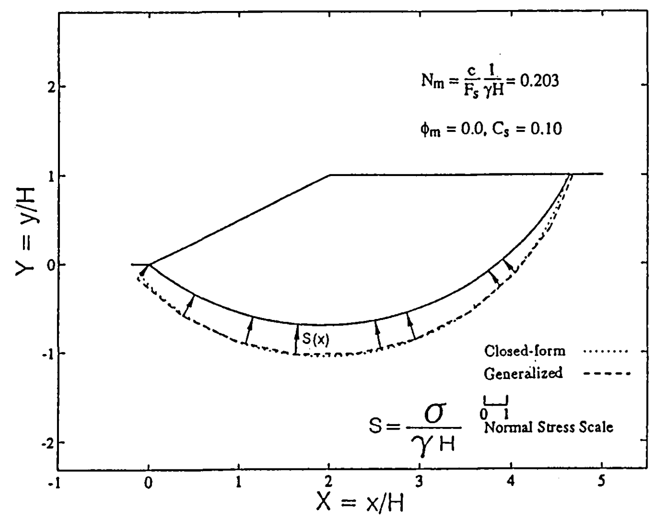

Fig. 5. Comparison between the closed-form approach and the generalized method: normal stress distribution (Case 3)

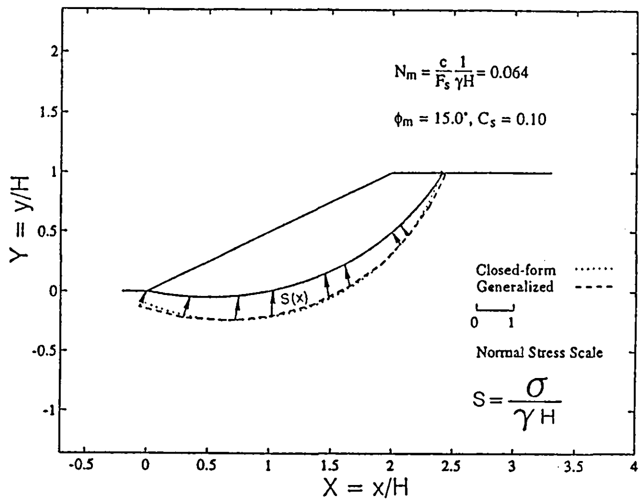

Fig. 6. Comparison between the closed-form approach and the generalized method: normal stress distribution (Case 4)

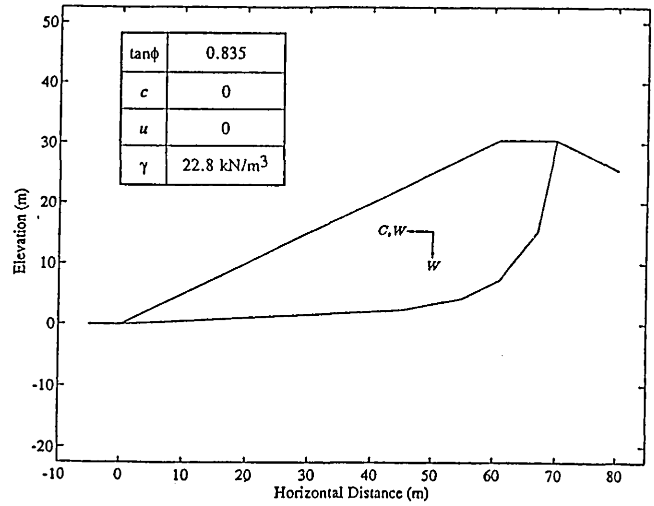

Fig. 7. The problem given by Sarma and Barbosa (1985)

Table 4. Critical values of C_s yielding F_s=1.0 for problem in Fig. 7

Method	Critical Value of C_s
Sarma (1973)	0.61
Morgenstern and Price (1965)	0.64
Sarma and Barbosa (1985)	0.58
Presented method	0.42

one obtained from the closed form solution is very good. Therefore, it appears that the generalized numerical scheme is reliable with respect to F_s and σ, the most important output in the engineering sense.

Figure 7 shows the problem presented by Sarma and Barbosa (1985). In this problem, the slip surface is of a general shape. Though it is a simple problem, it provides a comparison with three rigorous methods: Sarma (1973), Morgenstern and Price (1965) and Sarma and Barbosa (1985). Table 4 shows the comparison of the critical value of C_s (i.e., a value yielding $F_s = 1$) obtained by the different methods. The presented method yields the most critical value, i.e., the least value of C_s needed to bring the slope to a limit equilibrium state along the prescribed slip surface.

CONCLUSIONS

Extension of a generalized limit equilibrium approach to pseudo-static seismic slope stability analysis was introduced. Formulation of the extension together with a numerical scheme were briefly presented. It is demonstrated that for critical slip surfaces obtained from the closed form variational analysis, the generalized method (for the same surface) yields the same value of F_s. Comparison with other rigorous methods indicates the presented procedure yields a more critical F_s. However, this observation is limited to one problem. Therefore, further comparisons would be needed to draw out firm conclusions.

REFERENCES

1) Baker, R. and Garber, M.(1978): "Theoretical analysis of the stability of slopes," Géotechnique, Vol. 28, No. 4, pp. 395-411.
2) Leshchinsky, D. (1990): "Slope stability analysis: generalized approach," J. Geotech. Engrg., ASCE, Vol. 116, No. 5, pp. 851-867.
3) Leshchinsky, D. and Huang, C. C. (1992): "Generalized slope stability analysis: interpretation, modification and comparison," J. Geotech. Engrg., ASCE, Vol. 118, No. 10, pp. 1559-1576.
4) Leshchinsky, D. and San, K. C.(1993): "Pseudo-static slope stability analysis: design charts," J. Geotech. Engrg., ASCE, (in press).
5) Morgenstern, N. R. and Price, V. E., (1965): "The analysis of the stability of general slip surfaces," Géotechnique, Vol. 13, No. 1, pp. 79-93.
6) Sarma, S. K. (1973): "Stability analysis of embankments and slopes," Géotechnique, Vol. 23, No. 3, pp 423-433.
7) Sarma, S. K. (1979): "Stability analysis of embankments and slopes," J. Geotech. Engrg., ASCE, pp. 1513-1524.
8) Sarma, S. K. and Barbosa, M. R. (1985): "Seismic stability analysis for rockfill dams with central clay cores," Géotechnique, Vol. 35, No. 3, pp. 319-328.