Secant Young's modulus from N-value or c_u considering strain levels

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ABSTRACT: It has been pointed out that deformation moduli based on conventional in-situ and laboratory tests are generally too low to obtain appropriate deformations under design loads, and that shear moduli from seismic velocities G_d should be used as upper bounds on static shear moduli. However, direct measurements of G_d are expensive and cumbersome. Then it is often required to evaluate G_d from conventional tests in practice. Evaluation of G_d from SPT N-value or shear strength c_u is discussed, and the following relationships are recommended as average values: G_d=5N[MPa] or 500c_u. Introducing G_d or E_d as the initial tangents of simple hyperbolic curves, strain-dependent moduli for linear or simplified non-linear analyses are evaluated. As an example of application, pile settlement analysis with a simplified non-linear BEM is shown.

1 INTRODUCTION

Deformation analyses based on linear or simplified non-linear elastic models are practical and effective methods under design loads provided that appropriate input parameters are given. It has been recognized, however, that Young's moduli obtained from conventional in-situ or laboratory tests tend to largely overestimate deformations under design loads. This situation mainly arises from the fact that strain levels corresponding to the input parameters are often much larger than those in the field (e.g., Murayama et al. 1982, Jardine et al. 1985). Therefore, we should evaluate Young's moduli E or shear moduli G as a function of strain level; that is to say, appropriate moduli corresponding to in-situ strain levels should be given.

As is often the case in practice, however, available data could be only routine in-situ and/or laboratory tests such as Standard Penetration Tests (SPT) and unconfined compression tests. Consequently, we often have to make use of empirical correlations between SPT N-values or undrained shear strength c_u and E or G. Many such relationships have been proposed, but the strain levels in them are not definite except G_d based on shear wave velocities.

In this paper, evaluation of G_d or E_d from N-values or c_u is discussed. Then introducing E_d or G_d as the initial tangent of the Kondner or the Hardin-Drnevich hyperbolic curve, $E(\varepsilon)$ or $G(\gamma)$ is evaluated in terms of N-value or c_u. Application of these relationships to settlement analyses is also discussed.

2 HYPERBOLIC STRESS-STRAIN MODEL

There are a wide variety of sophisticated stress-strain models for soils. However, most of the models which require many parameters would be too difficult to use in practice, especially in preliminary analyses or projects which are not so big.

The isotropic linear elastic model needs only two parameters. If we can choose the appropriate parameters, it should be an adequate model for design (Poulos 1989). When average shear strain becomes on the order of 10^{-4}, or localized failure occurs, non-linear models are preferable (e.g., Bellotti et al. 1989).

In order to perform non-linear analyses, at least another parameter such as a strength parameter is required. The Kondner or the Hardin-Drnevich hyperbolic model is more advantageous than other non-linear models with the same number of parameters, such as the bilinear model, because of the following points (Hirayama 1990, 1991):

1. The model can simulate the essential non-linear features fairly well with the minimum number of parameters. If it is required to simulate other essential features according to a problem, other parameters may be introduced as will be shown in **4.1**.

2. The physical meanings of three parameters are clear; i.e., initial stiffness, ultimate strength, and Poisson's ratio.

3. The method for determining the constants of the hyperbola based on measured data is rather simpler in terms of calculation and judgment.

4. Theoretical analyses for interpreting the effects of soil non-linearity on foundation behaviour are possible in some cases since the closed-form solutions for integrals under simple boundary conditions may be obtainable as an example will be shown in **4.1**.

As mentioned above, three input parameters are required. The ultimate or peak strength, which is the primary parameter in conventional failure analyses, can be easily determined from the results of routine tests.

Poisson's ratio under undrained conditions is definite. In the case of drained conditions, Poisson's ratio has to represent dilatancy characteristics. Consequently, non-linear elastic models cannot represent positive dilatancy. If positive dilatancy is important in a problem analyzed, such as a shallow foundation on dense sand or over-consolidated clay, elastic models may be difficult to predict overall deformation behaviour; e.g., both settlements and lateral displacements. It should be also noted that bulk modulus is very sensitive to Poisson's ratio. Therefore, Poisson's ratio is very important when deformation mechanisms mainly depend on volume change rather than shear deformation such as pile toe behaviour under relatively large settlement (Hirayama 1991). In such cases, linear or non-linear elastic analyses should be interpreted with caution acknowledging the above-mentioned limitation and mechanism.

Initial stiffness such as initial Young's modulus or initial shear modulus should be given considering the derived actual strain level of soil. It should be noted that so-called initial stiffness derived from conventional tests, such as traditional triaxial tests and pressuremeter tests, is too low in general. This will be discussed in detail in the following section.

3 EVALUATION OF INITIAL AND SECANT MODULI FROM CONVENTIONAL TESTS

3.1 Initial shear modulus from shear wave velocity, G_a

Shear moduli obtained by seismic techniques, where imposed shear stresses are much smaller than those by foundations, used to be considered unpractical for static loading problems. It has shown, however, that both static shear moduli and seismic shear moduli are identical when precise measurements are made at very low shear strain levels of the order of 10^{-5} (Drnevich 1975). Then he proposed that shear moduli in-situ by seismic techniques be used as upper bounds on static shear moduli. Tatsuoka & Shibuya (1991) have recently reviewed previous studies, and commented that the results strongly support the methodology which is the same with the above-mentioned proposal.

Therefore, shear modulus from shear wave velocity G_a, whose strain level is usually about 10^{-6} or less (Ohsaki & Iwasaki 1973, Hara et al. 1974), can be one of the most important parameter in static analyses as well as dynamic analyses for earthquake problems.

However, direct measurements of G_a in-situ and in the laboratory are expensive and cumbersome. Then various empirical correlations between G_a and common test results have been proposed in the field of earthquake engineering. Use of these correlations may be recommended in practice in some cases as follows (e.g., Gazetas 1991):

1. in feasibility studies and preliminary design calculations;
2. for final design calculations in big projects as supplementary data or in small projects as main data;
3. for initial data in back analyses;
4. to provide an order-of-magnitude check against the experimentally determined values.

In the following subsections, evaluation of G_a from SPT N-value and undrained shear strength c_u will be discussed.

3.2 G_a from N-value

Various correlations between G_a and SPT N-value have been proposed in Japan and the USA. Some of them take account of factors or parameters such as soil type, geologic age and effective vertical stress. Sykora & Koester (1988) investigated existing correlations and commented that correlations involving these factors or parameters do not improve accuracy so much or at all.

Therefore, the following correlations which involve various soils are discussed:

$$G_d = 11.9N^{0.78} \quad [\text{Mpa}] \qquad (1)$$

$$G_d = 14.1N^{0.68} \quad [\text{MPa}] \qquad (2)$$

Eq.(1) was proposed by Ohsaki & Iwasaki (1973) from about 220 data, and Eq.(2) was proposed by Imai & Tonouchi (1982) from 1654 data. These data can be considered to cover representative ground conditions in urban areas of Japan.

Both correlations were obtained by statistic analyses. The methods of analyses were not described, but it is inferred that they are based on the method of least squares employing N-value as an independent variable, which should not involve errors from the assumption of the method. However, if G_d and N-value involve the same order of errors, the correlation between them should be based on the principal component analysis as illustrated in **Fig.1** (Hirayama et al. 1988).

In $G_d=CN^n$, the constants obtained from the principal component analysis give smaller C and larger n than Eqs.(1) or (2). Then from **Fig.2**, Eqs.(1) or (2) may be approximated by

$$G_d = 5N \quad [\text{MPa}] \qquad (3)$$

If it is assumed that SPT N-values measured in Japan correspond to an efficiency of 67% of free-fall energy (N_{67}) and those in the USA correspond to 60% (N_{60}) (Sykora & Koester 1988), the correlation for sands and silty sands proposed by Seed et al. (1983) may be adjusted as follows:

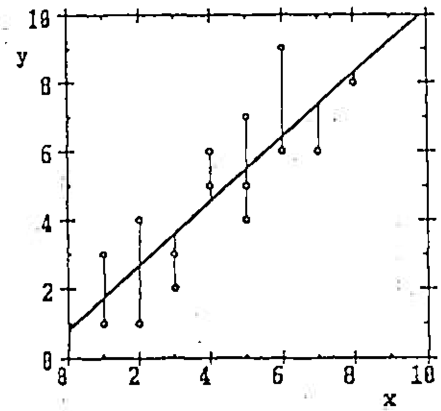
$$G_d = 6.2N_{60}[\text{MPa}] = 5.6N_{67}[\text{MPa}] \qquad (4)$$

Eq.(3), which is simpler than Eq.(1) or (2), is in good agreement with Eq.(4).

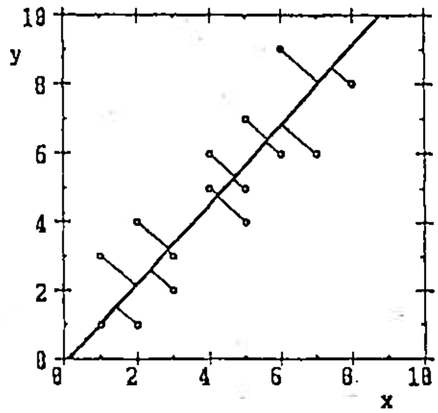
3.3 G_d from c_u

Errors of G_d from SPT become large when N-values are not so large, especially when N-value is less than 2 (Hara et al. 1974, Imai 1977). For such soft clays, G_d from shear strength c_u could be more reliable.

Shibata & Soelarno (1978) discussed that G_d/c_u might be constant when C_c is about 0.75 and high plastic clays have lower G_d/c_u. Tatsuoka & Shibuya (1991) pointed out that G_d/c_u decreases as c_u increases.



(a) method of least squares



(b) principal component analysis

Fig.1 Methods of statistic analysis

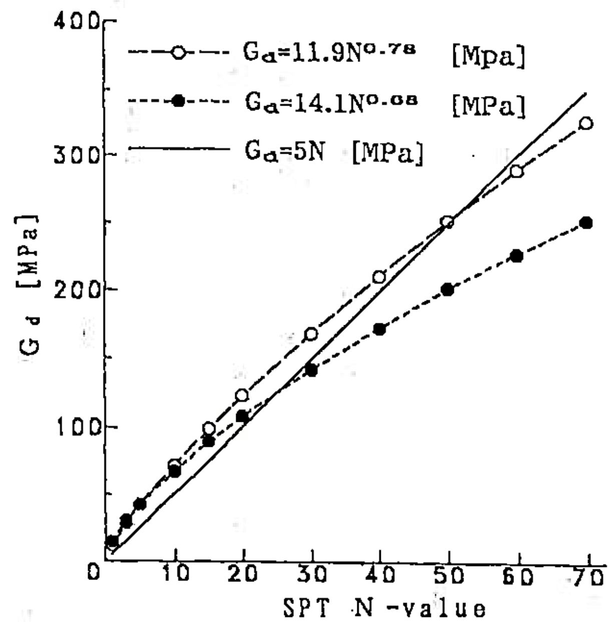


Fig.2 Comparison of Eqs.(1), (2) & (3)

Thus the values of G_d/c_u could take a rather wide range. Hara et al. (1974) investigated the correlation between G_d and c_u. The values of G_d/c_u range from 250 to 1430, but a correlation which is approximated with the following equation is obtained with a high coefficient of correlation:

$$G_d=500c_u \quad (5)$$

Several other correlations have been proposed, e.g., as summarized in Hara et al. (1974) and Shibata & Soelarno (1978). Some of them give a bit higher G_d than Eq.(5). Thus Eq.(5) may give appropriate average values which is safe sides.

3.4 Secant moduli

Evaluation of G_d from N-value or c_u is discussed in **3.2** and **3.3**. Eqs.(3) and (5) are recommended for the purposes summarized in the last part of **3.1**.

When Young's modulus under very low strain which corresponds to G_d is required, the following relationships are recommended from Eqs.(3), (5), and dynamic Poisson's ratios shown by Ohsaki & Iwasaki (1973) (i.e., 0.48 for cohesive soils and 0.44-0.46 for various soils):

$$E_d=14N\ [\mathrm{MPa}] \quad (6)$$

$$E_d=1500c_u \quad (7)$$

The reference strain in the Hardin-Drnevich hyperbolic curve γ_r ([asymptote value]/[initial tangent]) may be assumed in rather simple forms as follows: $\gamma_r=10^{-2.5}\ \sigma_c^{0.5}$ (σ_c = confining pressure in [MPa]) for cohesionless soils (Shibata & Soelarno 1975) and γ_r=1/500 for cohesive soils (from Eq.(5)). Then secant moduli taking account of strain levels are evaluated as follows (Hirayama & Fukuda 1988):

$$G_{sec}(\gamma)=5N/(1+10^{2.5}\ \sigma_c^{-0.5}\ \gamma)[\mathrm{MPa}] \quad (8)$$

$$G_{sec}(\gamma)=500c_u/(1+500\ \gamma) \quad (9)$$

$$E_{sec}(\varepsilon)=14N/(1+1.4\mathrm{x}10^{2.5}\ \sigma_c^{-0.5}\ \varepsilon)[\mathrm{MPa}] \quad (10)$$

$$E_{sec}(\varepsilon)=1500c_u/(1+750\ \varepsilon) \quad (11)$$

where σ_c=confining pressure in [MPa].

Eq.(6) gives just 20 times as large as E_p=0.7N[MPa], which was proposed based on conventional pressuremeter tests (Yoshinaka 1968) and has been widely used in Japan. This is due to the fact that strains correspond to E_p are in a range of 10^{-2} to 10^{-1} (Imai 1977). Eq.(10) can explain the above-mentioned E_p when strain levels in pressuremeter tests are taken into account.

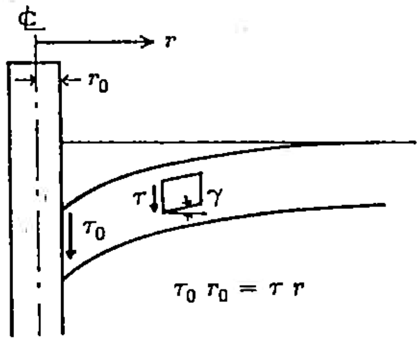


Fig.3 Concentric-cylinder model for soil behaviour around a pile shaft

4 APPLICATION TO SETTLEMENT ANALYSIS

4.1 Three-constant hyperbolic model for pile settlement analysis

Soil behaviour around a pile shaft, whose radius is r_o, is modeled with a concentric-cylinder model as shown in **Fig.3**. Stress-strain behaviour of soil is modeled with the three-constant (except Poisson's ratio) hyperbolic curve as shown in **Fig.4**. Initial tangent shear modulus G_i is practically the same with G_d. Another parameter R_f which is the ratio of peak strength to asymptote value is introduced into the two-constant hyperbolic curve described in **2**, but it should be noted that the three-constant model reduced to the two-constant model when R_f is 1.0. Introducing R_f herein is rather for comparing with the result derived later. Secant shear modulus at a distance of r from the center, $G_{sec}(r)$, is given by

$$G_{sec}(r)=\tau/\gamma=1/(a+b\gamma)=G_i[1-(r_o/r)F] \quad (12)$$

where $F=\tau_o/\tau_{ult}$ (=load factor).

Then, the average soil modulus from r_o to mr_o, $G_{sec,av}$, is calculated as follows (Hirayama 1991):

$$G_{sec,av}=G_i[1-2F/(1+m)] \quad (13)$$

Eq.(13) indicates that the average stress-strain response from r_o to mr_o is also a three-constant hyperbola whose initial modulus is the same with Eq.(12) (i.e., $G_i=1/a=G_d$) while the asymptote is (1+m)/2 times as large as that of Eq.(12) at the interface (i.e, r_o/r = 1) whose shear strain is largest. That is to say, R_f equals 2/(1+m) for average stress-

$$\tau = \frac{\gamma}{a + b\gamma} \quad \text{with} \quad \tau \leq \frac{R_f}{b}$$

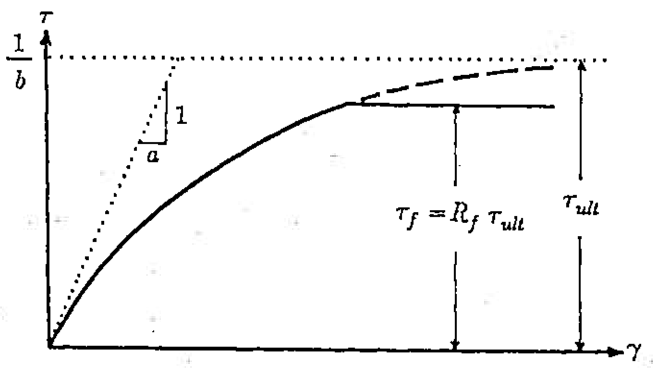


Fig.4 Three-constant stress-strain model

Table 1 Input parameters

c_u[kPa]=53+7.2z[m]
E_i=1500c_u
f_s=0.5c_u (unit skin friction)
q_{ult}=9c_u (ultimate unit toe resistance)
ν =0.5 (Poisson's ratio)
m=7 (Eq.(13))
E_{pile}=2.78x10^4[MPa]

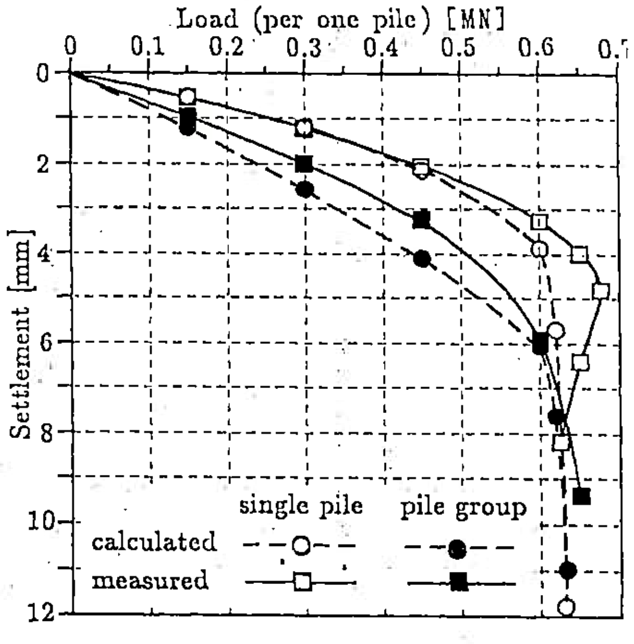


Fig.5 Calculated and measured load-settlement curves

strain behaviour around a pile shaft when R_f=1.0 (i.e., perfectly hyperbolic) for soil itself. The apparent increase of stiffness due to R_f=2/(1+m) arises from the contribution of the surrounding soil, whose shear strain decreases as the distance from the interface increases as shown in Eq.(12), on shaft resistance as far as continuous slip does not occur at or near pile-soil interface. Part of Eq.(13) up to the strength at the interface may be used for pile settlement analysis. Average soil moduli between piles, which may be used for pile-soil-pile interaction for pile group settlement analyses, are also approximately calculated in closed-form solutions (Hirayama 1991). Then simple but reasonable non-linear analyses for piles or pile groups can be carried out, and a program termed PIESAVR for a simplified BEM which is used with personal computers has been written (Hirayama 1991, 1992).

4.2 Case study of pile settlement analysis

The loading tests by O'Neill et al. (1982) for 3x3 pile group and two single piles, which were 13m long steel tube piles driven in heavily overconsolidated clays, were analyzed with PIESAVR. Input parameters determined from the proposals described in **3.4** are shown in **Table 1** (Hirayama 1991).

The results of calculated load-settlement relationships are shown in **Fig.5** compared with the measured results. The calculated results agree fairly well with the measured results. When the effects of pile installation such as pre-boring up to 3m and initial residual stresses due to driving and re-consolidation are considered, the calculated results are improved (Hirayama 1992).

As another example, Fuse et al. (1992) analyzed the result of a loading test of a floating pile at Kansai International Airport, which is under construction in Osaka Bay, with FEM employing a elasto-viscoplastic model. They concluded that initial moduli from Eqs.(6) & (7) give much better agreement than other lower moduli.

4.3 Linear elastic analysis

Secant moduli may be evaluated from Eqs.(8)-(11) when strain levels are known. Then linear analyses taking account of strain levels are possible. For example, in the case of settlement analyses of shallow foundations, simplified strain distribution patterns such as the 2B-0.6 distribution or its improved distribution (Schmertmann et al. 1978) may be used.

However, strain levels are not known until settlements are calculated. First, therefore, strain levels have to be assumed. The strain levels from calculated settlements should be the same order with the assumed strain levels. Few feedback calculations will give reasonable agreement.

It may be useful to know the normal range of strain levels in the field, such as 10^{-4} to 10^{-3} (about 3 times for clays) for strutted excavation problems (Hirayama & Fukuda 1988). Jardine et al. (1986) analyzed some problems; such as shallow foundation, pile and strutted excavation; with a non-linear FEM, and summarized the secant moduli with load factor or safety factor.

5 CONCLUSIONS

Deformation moduli from shear wave velocities G_d or E_d, which can be used as upper bounds on static analyses, are evaluated in terms of conventional in-situ and laboratory test results; i.e., SPT N-value and shear strength c_u. The following relationships are recommended as average values: G_d=5N[MPa] or $500c_u$, E_d=14N[MPa] or $1500c_u$.

Introducing these relationships into the Hardin-Drnevich or the Kondner hyperbolic curve, secant moduli of G or E are evaluated taking account of strain levels as shown in Eqs.(8) to (11). These relationships can be effectively used for linear or simplified non-linear analyses.

REFERENCES

Bellotti,R., V.Ghionna, M.Jamiolkowski, P.K.Robertson & R.W.Peterson 1989. Interpretation of moduli from self-boring pressuremeter tests in sand. Geotechnique 39(2): 269-292.

Drnevich,V.P. 1975. In-situ measurement of initial stresses and deformation characteristics. Proc. ASCE Specialty Conf. on In-situ measurement of soil properties, Vol.2: 244-252.

Fuse,Y., N.Takahashi, T.Mizuta & E.Kou 1992. Analysis of shaft friction of a floating pile in young reclaimed ground (Kansai International Airport). Proc. 27th Japan National Conf. on SMFE: 1477-1478 (in Japanese).

Gazetas,G. 1991. Foundation vibrations. In H.-Y.Fang (ed.), Foundation engineering handbook 2nd ed.: 553-593. New York: Van Nostrand Reinhold.

Hara,A., T.Ohta, M.Niwa, S.Tanaka & T.Banno 1974. Shear modulus and shear strength of cohesive soils. Soils and Foundations 14(3): 1-12.

Hirayama,H. 1990. Load-settlement analysis for bored piles using hyperbolic transfer functions. Soils and Foundations 30(1): 55-64.

Hirayama,H. 1991. Pile-group settlement interaction considering soil non-linearity. Computer methods and advances in geomechanics: 139-144. Rotterdam: Balkema.

Hirayama,H. 1992. Analysis of residual stresses in driven piles using a non-linear simplified BEM. Piling: European practice and worldwide trends: 72-77. London: Thomas Telford.

Hirayama,H. & E.Fukuda 1988. Estimation of the secant modulus of elasticity from SPT N-values taking account of strain levels of soils. Proc. 23rd Japan National Conf. SMFE: 1247-1250 (in Japanese).

Hirayama,H., K.Hayakawa & S.Sawada 1988. Linear models for correlation between two variables -principal component analysis and method of least squares. Proc. 23rd Japan National Conf. SMFE: 1251-1252 (in Japanese).

Imai,T. 1977. P- and S-wave velocities of the ground in Japan. Proc. 9th ICSMFE, Vol.2: 257-260.

Imai,T. & K.Tonouchi 1982. Correlation of N value with S-wave velocity. Proc. 2nd European Symp. on Penetration Testing: 67-72.

Jardine,R.J., A.Fourie, J.Maswoswe & J.B.Burland 1985. Field and laboratory measurements of soil stiffness. Proc. 11th ICSMFE, Vol.2: 511-514.

Jardine,R.J., D.M.Potts, A.B.Fourie & J.B.Burland 1986. Studies of the influence of non-linear stress-strain characteristics in soil-structure interaction. Geotechnique 36(3): 377-396.

Murayama,S., Y.Iwasaki, T.Hashimoto, H.Hirayama & J.Matsuzaki 1982. Ground heave due to the excavation and observational construction control. Tsuchi-to-kiso, Japanese Soc. of SMFE 30(7): 31-36 (in Japanese).

Ohsaki,Y. & R.Iwasaki 1973. On dynamic shear moduli and Poisson's ratios of soil deposits. Soils and Foundations 13(4): 61-73.

O'Neill,M.W., R.A.Hawkins & L.J.Mahar 1982. Load transfer mechanisms in piles and pile groups. J. Geotech. Eng., ASCE 108(12): 1605-1623.

Poulos,H.G. 1989. Pile behaviour -theory and application. Geotechnique 39(3): 365-415.

Schmertmann,J.H., J.P.Hartman & P.R.Brown 1978. Improved strain influence factor diagrams. J. Geotech. Eng., ASCE 104(8): 1131-1135.

Seed,H.B., I.M.Idriss & I.Arango 1983. Evaluation of liquefaction potential using field performance data. J. Geotech. Eng., ASCE 109(3): 458-481.

Shibata,T. & D.S.Soelarno 1975. Stress-strain characteristics of sands under cyclic loading. Proc. JSCE, No.239: 57-65 (in Japanese).

Shibata,T. & D.S.Soelarno 1978. Stress-strain characteristics of clays under cyclic loading. Proc. JSCE, No.276: 101-110 (in Japanese).

Sykora,D.W. & J.P.Koester 1988. Correlations between dynamic shear resistance and standard penetration resistance in soils. Earthquake engineering and soil dynamics II -Recent advances in ground-motion evaluation, G.S.P. No.20: 389-404. New York: ASCE.

Tatsuoka,F. & S.Shibuya 1991. Deformation characteristics of soils and rocks from field and laboratory tests. Proc. 9th Asian Reg. Conf. SMFE, Vol.2: 53-114.

Yoshinaka,R. 1968. Coefficient of lateral subgrade reaction. Doboku-gijyutsu-shiryou 11(1): 32-37 (in Japanese).