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Determination of shear wave velocity structures from spectrum analyses of short-period microtremors

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ABSTRACT: Use of short-period microtremors is studied for the evaluation of shear wave velocity (V_s) structures. Vertical ground surface motions of short-period microtremors are observed using circular arrays of six vertical sensors. Based on high-resolution frequency-wavenumber (F-K) spectrum analysis or the spacial auto-correlation analysis of these data, Rayleigh wave dispersion characteristics can be determined. The obtained dispersion curves are consistent with those determined by the steady-state Rayleigh wave method. An inverse analysis using these dispersion data permits estimating V_s structures of near surface soils. The V_s structures inferred by the proposed method show fairly good agreement with those obtained by the conventional down-hole method. These findings indicate that the use of short-period microtremors is effective for estimating V_s structures of near surface soils.

INTRODUCTION

The use of microtremors has been extensively studies for the evaluation of dynamic response characteristics of sites since the pioneer works by Kanai and Tanaka (1961). There are essentially two methods to use microtremors for this purpose, i.e., the spectrum method and the phase velocity method.

These two methods contrast well in their test procedures and analyses. The spectrum method measures microtremors at a point using one three-component sensor. The site characteristics are then estimated based on such factors as the predominant period which is determined from the Fourier spectrum analysis of horizontal motions of microtremors. The major problem associated with this method is the effects of unidentified sources on the measured spectrum. Consequently, the spectrum peak may not always reflect the response characteristics of the site but the exciting function of the source (Finn 1991).

The phase velocity method observes simultaneously the vertical motions of microtremors using arrays of sensors. The V_s structure of the site is estimated through spectrum analyses of these motions (Asten and Henstridge 1984; Horike 1985; Okada and Matsushima 1986). The principle of this method lies in the facts that microtremors consist of predominant Rayleigh waves and that spectrum analyses of microtremors can yield dispersion characteristics of Rayleigh waves which reflect the V_s structure of the site. Unlike the other surface wave methods such as the SASW method (e.g., Nazarian and Stokoe 1984), this method does not require to actively generate surface waves. Thus, the method may be called passive Rayleigh wave method. This method consists of three steps: (1) the measurement of microtremors using arrays of sensors, (2) the determination of a dispersion curve from spectrum analysis of the observed motions, and (3) the inverse analysis of the dispersion curve.

There are two methods to extract dispersion characteristics from microtremor data, i.e., the frequency-wavenumber (F-K) spectrum analysis (e.g., Capon 1969) and the spacial auto-correlation analysis (e.g., Aki 1957). From a theoretical point of view, the later method is valid only in the cases where observed waves are unidirectional or isotropic. However, with this analysis, Okada and Matsushima (1986) have determined Rayleigh wave dispersion curves based on microtremor measurements. Although only applicable to data measured with a circular array, this analysis can readily be conducted with a personal computer, thereby enhancing the performance and reliability of the field test.

The passive method has been successfully applied to the determination of deep soil structures based on the measurements of long-period microtremors in the period range generally longer than 1 s (e.g., Asten and Henstridge 1984; Horikc 1985; Okada and Matsushima 1986). Despite the good application of the method to deep soils, it has seldom been used for characterizing V_s structures of near surface soils which requires the measurements and analyses of short-period microtremors. This is partly because higher-mode Rayleigh waves, which tend to dominate in this period range, make both reliable determination and inverse analysis of dispersion curves difficult (Tokimatsu et al. 1992a).

Recently, Tokimatsu et al. (1992a) have successfully determined Rayleigh wave dispersion curves and V_s structures based on the F-K spectrum analysis of short-period microtremors. However, more field data are required to confirm the effectiveness of the method. The objects of this report are to present the results of additional field tests and to discuss the possible use of short-period microtremors for V_s profiling of near surface soils. Also discussed is the applicability of the spacial auto-correlation analysis to the determination of phase velocities of Rayleigh waves in the short-period range.

DETERMINATION OF DISPERSION CHARACTERISTICS AND SHEAR WAVE STRUCTURES

Field Test Procedures

The test procedures and equipments used in the field are the same as those described by Tokimatsu et al. (1992a). The test equipment consists of six sensors, amplifiers, and a laptop computer. The sensors are vertical velocity transducers with a natural period of 1 s. The computer is a model PC-386LS from EPSON, equipped with an A/D converter having a resolution of 12 bits.

Six vertical sensors are placed on the ground surface to form a circular array with a sensor in the center (Fig. 1). The radius of the array is originally set at 5 meters and expanded or contracted by a factor of about two so that the range of array radius covers the range of wavelengths to be measured (Asten and Henstridge 1984).

The vertical ground surface motions of microtremors are observed with each array. The analog motions measured with the sensors are amplified, converted into digitized form, and then stored in the hard disk of the computer. When the signal-to-noise (S/N) ratios of the measured motions are low, random vibrations are generated artificially with a known distance from the array by tapping the ground surface by a hammer or a foot to provide adequate S/N ratios over the frequency range of interest. This situation often occurs in the frequency range over 10 Hz.

Spacial Auto-Correlation Analysis

The phase velocity of Rayleigh waves may be determined by the spacial auto-correlation analysis of microtremor data measured with a circular array (Aki 1957; Okada and Matsushima 1986). Assuming that M sensors are available and that the *M*th sensor is the one located at the center of the array, the phase velocity at a frequency f, c, may be defined by:

$$\rho_{ave} = J_0\left(\frac{2\pi f}{c} r\right) \quad (1)$$

in which J_0 is the Bessel function of the first kind of the order zero, r is the radius of the array, and ρ_{ave} is the azimuthally averaged auto-correlation coefficient defined by

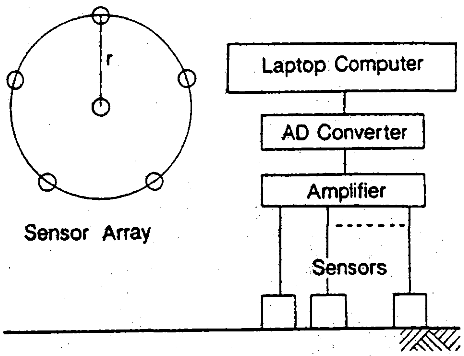

Fig. 1 Schematic diagram of test arrangement

$$\rho_{ave} = \sum_{i=1}^{M-1} \phi(f,i)/\phi(f,M) \quad (2)$$

in which $\phi(f,i)$ is the spacial auto-correlation function of the waves between the *i*th and *M*th sensors which are filtered by a resonator of the frequency f. By knowing the array radius and the spacial auto-correlation coefficient given by Eq. (2), the phase velocity can then be determined from Eq. (1). The corresponding wavelength can then be given by

$$\lambda = c/f \quad (3)$$

By repeating the above computation over the frequency range of interest, a dispersion curve may be obtained.

Frequency-Wavenumber Spectrum Analysis

The phase velocity of Rayleigh waves can also be determined through the high-resolution frequency-wavenumber (F-K) spectrum analysis developed by Capon (1969). Assuming that M sensors are available and that the *i*th sensor is located at the vector position $\mathbf{x}_i$, the high-resolution F-K spectrum, P(f,**k**), is defined by

$$P(f,\mathbf{k}) = \sum_{i=1}^{M} \sum_{j=1}^{M} A_i^*(f,\mathbf{k})A_j(f,\mathbf{k})G_{ij}(f)\exp[i\mathbf{k}\cdot(\mathbf{x}_i-\mathbf{x}_j)] \quad (4)$$

where * denotes complex conjugate, f is the frequency; **k** is the vector wavenumber in cycles per meter;

$$G_{ij}(f) = \frac{1}{N}\sum_{n=1}^{N} S_{in}(f)S_{jn}^*(f) \quad (5)$$

in which N is the total number of the nonoverlapping data segments, and S_{in} is the Fourier transform of the data in the *i*th sensor and in the *n*th segment; and

$$A_i(f,\mathbf{k}) = \sum_{j=1}^{M} q_{ij}(f,\mathbf{k}) / \sum_{i=1}^{M}\sum_{j=1}^{M} q_{ij}(f,\mathbf{k}) \quad (6)$$

in which $\{q_{ij}(f,\mathbf{k})\}$ is the inverse of the matrix $\{\exp[i\mathbf{k}\cdot(\mathbf{x}_i-\mathbf{x}_j)]G_{ij}(f)\}$.

The high-resolution F-K spectrum is drawn on a two-dimensional wavenumber (k_x-k_y) space for each frequency. The peak of this F-K spectrum provides the information concerning the phase velocity of propagating waves. If a peak occurs at a distance of $|\mathbf{k}|$ from the origin at a frequency f, the corresponding phase velocity, c, and the wavelength, λ, can be given by

$$c = 2\pi f/|\mathbf{k}| \quad (7)$$

$$\lambda = 2\pi/|\mathbf{k}| \quad (8)$$

By repeating the above computation over the frequency range for all data from different array diameters, a dispersion curve can be obtained.

Determination of Shear Wave Structures

The estimation of shear wave velocity of a deposit requires an inverse analysis using the observed dispersion data. Based on the studies by Haskell (1953) and Harkrider (1964), Tokimatsu et al. (1992b) have defined an apparent dispersion curve for multiple-mode Rayleigh waves that propagate along the surface of a horizontally stratified soil layer consisting N sublayers. This theoretical dispersion curve can readily be incorporated into the inverse analysis. The inverse process is then to minimize the misfit of the observed dispersion data with the theoretical dispersion curve.

Since the mass density and P-wave velocity of each layer have insignificant effects on the theoretical dispersion curve, these two parameters are predetermined and the S-wave velocity and thickness of each layer are sought in the inversion. The details of the inverse process and the consideration of higher modes have been described by Tokimatsu et al. (1992a).

COMPARISON OF SHEAR WAVE STRUCTURES OBTAINED BY VARIOUS METHODS

To evaluate the effectiveness of the short-period microtremors for the determination of V_s profiles, field tests were made at two sites: one at Kamidokoro, Niigata city; and the other Daiba, Tokyo, Japan; and the results are compared with available data.

Kamidokoro Site

The geological and geophysical logs at Kamidokoro determined from the conventional method (Tokimatsu

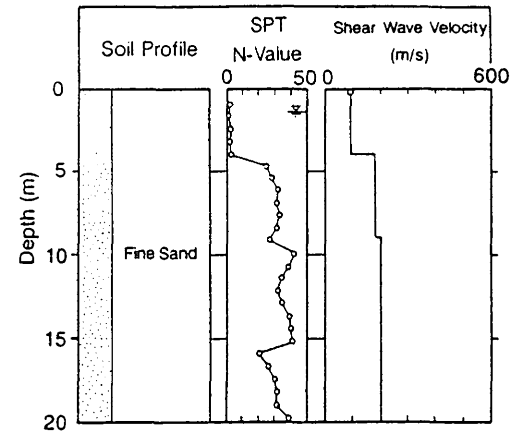

Fig. 2 Soil profiles at Kamidokoro Site

et al. 1991) are shown in Fig. 2. The layer from the ground surface to 4 m depth is loose sand which is underlain by a layer of dense sand. The shear wave velocities at depths less than 4 meters are less than 100 m/s, whereas they are more than 180 m/s at depths greater 4 meters. The maximum radius of the array used was 30 m.

Fig. 3 shows high-resolution F-K spectra on a two-dimensional wavenumber space at several frequencies for short-period microtremors (Fig. 3(a) to Fig. 3(c)) and for forced vibrations (Fig. 3(f)). The spectra are drawn as contours of $-10\log[P(f,k)/P_{max}(f)]$ in which $P_{max}(f)$ is the maximum value of $P(f,k)$. The maximum of the spectral power is indicated by an asterisk and the contours of the spectral power are drawn from 0 to 12 dB in steps of 2 dB. The direction at which a peak

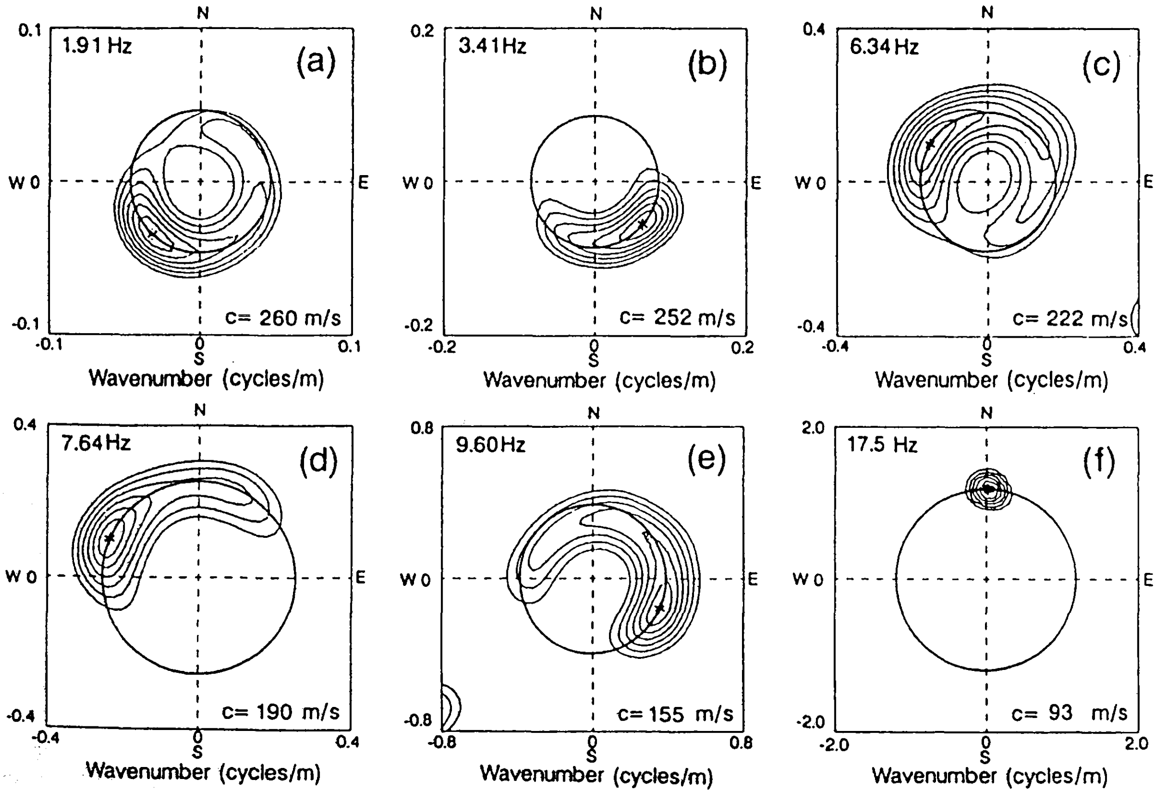

Fig. 3 F-K spectra for microtremors at Kamidokoro

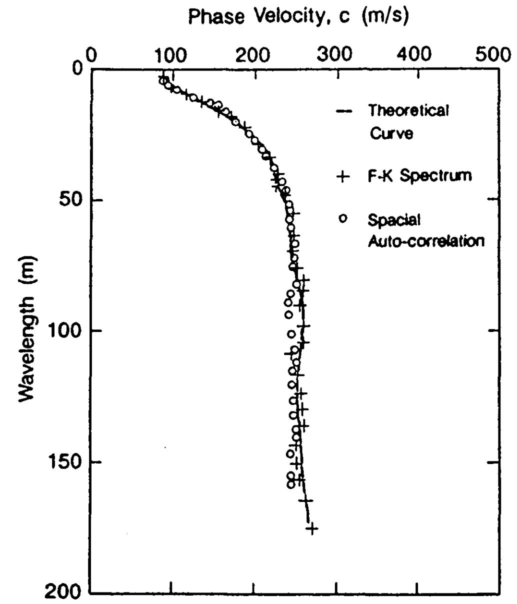

Fig. 4 Observed and computed dispersion curves at Kamidokoro

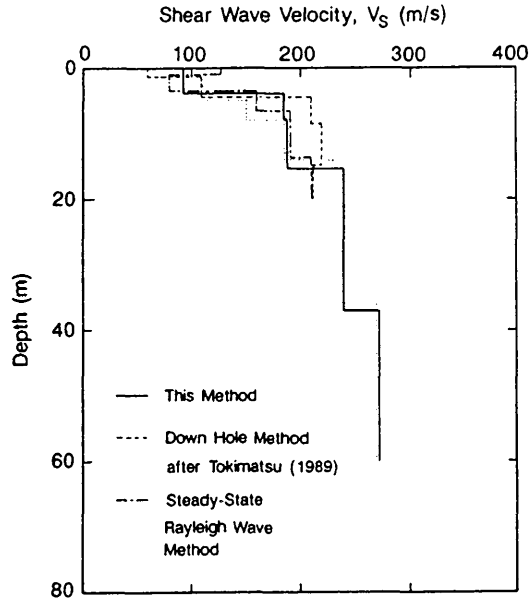

Fig. 6 V_s profiles obtained by different methods at Kamidokoro

occurs varies with frequency, indicating that the microtremors over the frequency range from 1 to 10 Hz propagate from various directions. This is because the sources of microtremors are distributed around the site as it is located within the city. The appearance of a sharp peak on the positive k_y axis in Fig. 3 (f) is simply because the forced vibrations were generated on the north of the array.

The dispersion curve determined by the F-K spectral analysis is shown in Fig. 4 in crosses. The data show a normally dispersive trend in which the phase velocity increases with increasing wavelength. Also shown in the figure in open circles is the dispersion curve determined by the spacial auto-correlation method. It appears that the both methods yield the same dispersion curve.

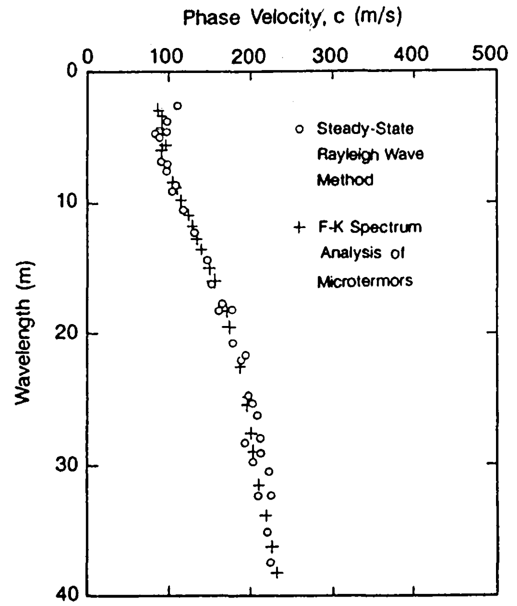

Fig. 5 Dispersion curves obtained by different methods at Kamidokoro

Fig. 5 compares the dispersion curve obtained from the F-K spectrum method with that from the steady-state Rayleigh wave method (Tokimatsu et al. 1991). The dispersion curves obtained by the two methods agree reasonably well for wavelengths up to 40 m. Because of the difficulty in generating surface waves with long wavelengths, the steady-state method can determine a dispersion curve only for wavelengths shorter than 40 m. In contrast, since the microtremors contain Rayleigh waves with long wavelengths, the dispersion characteristics can be determined over a wide range of wavelength. This is one of the significant advantages of the passive Rayleigh wave method.

The inverse analysis is conducted for the observed dispersion data assuming a soil layer model with appropriate initial properties. The dispersion curve for the inverted soil layer model is shown in Fig. 4 in a solid line, which is compatible with the observed data.

Fig. 6 shows the V_s profile estimated from the inversion. The shear wave velocity of the top layer is approximately 100 m/s to a depth of 4 m, which is underlain by a layer with V_s varying from 180 to 270 m/s to a depth of 50 m. Also shown in the figure are the V_s structures determined by the down-hole method (Fig. 2) and by the steady-state Rayleigh wave method (Tokimatsu et al. 1991). Unfortunately, the V_s profile deeper than 20 m for the site is unknown, but Tokimatsu (1989) has reported the soil profile near the site in which the shear wave velocity at depths between 20 and 70 m increases from 230 to 270 m/s with depth. Thus, the V_s structure estimated from the spectrum analyses of short-period microtremors appears to be consistent with those obtained by the other methods.

Daiba Site

The geological and geophysical logs near Daiba site determined from the conventional method are shown in Fig. 7. The layers from ground surface to 30 m depth consist of sandy clay and clay with V_s = 150 m/s.

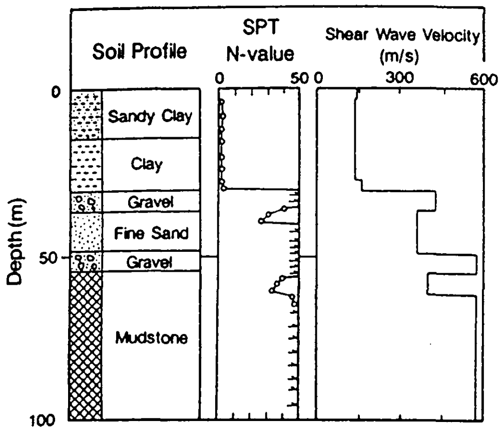

Fig. 7 Soil profiles at Daiba

These layers are underlain by layers of dense gravel and fine sand to a depth of 55 m. The average shear wave velocity of these layers is about 400 m/s. Underlying these layers is mudstone with V_s over 500 m/s. The maximum radius of the array used was 30 m.

Fig. 8 shows high resolution F–K spectra on a two-dimensional wavenumber space at several frequencies for the short-period microtremors. In each figure, a peak appears on the south, indicating that the microtremors over the frequency range from 1 to 10 Hz propagate mainly from that direction. This is probably due to heavy traffic on a highway running from the east to the west on the south of the site.

Fig. 9 shows the dispersion characteristics resulting from the F–K spectral analysis and the spacial auto-correlation method for all data sets. Again the results

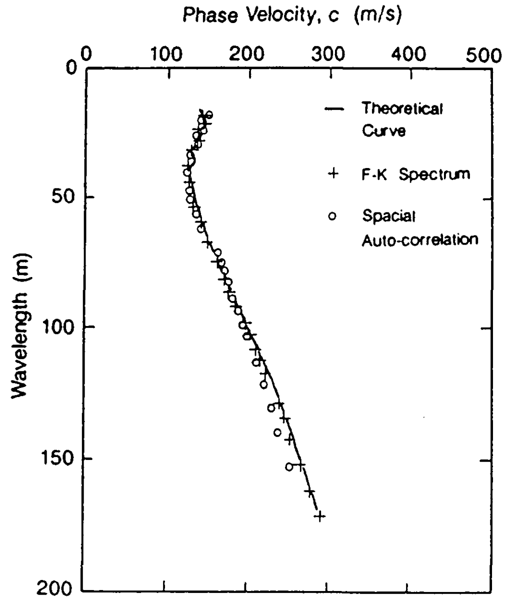

Fig. 9 Observed and computed dispersion curves at Daiba

of the two methods are in good agreement with each other. The data show an inversely dispersive trend at wavelengths shorter than 50 m. The inverse analysis is conducted for the observed dispersion data assuming a soil layer model with appropriate initial properties. The theoretical dispersion curve from the inverse analysis is shown in Fig. 9 in a solid line, which is compatible with the observed data. Note that the inversely dispersive trend observed in the measured data is well simulated by the theoretical dispersion curve. This simulation can

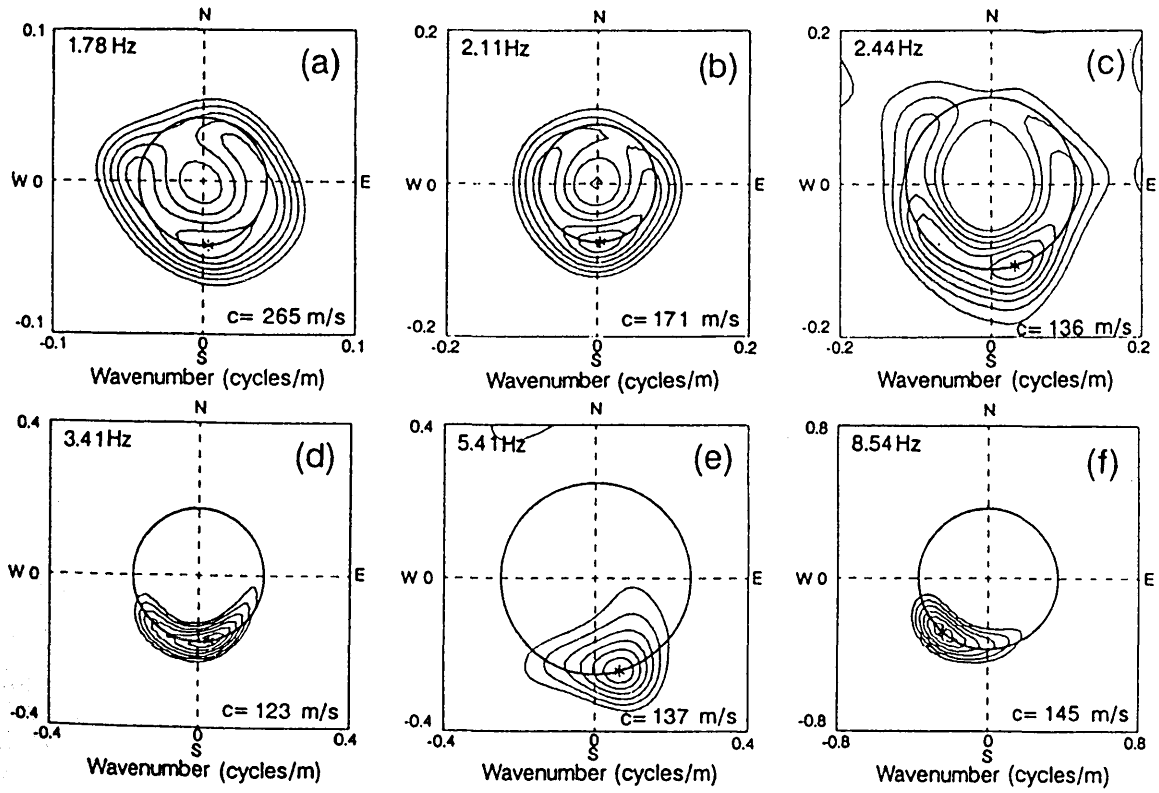

Fig. 8 F–K spectra for microtremors at Daiba

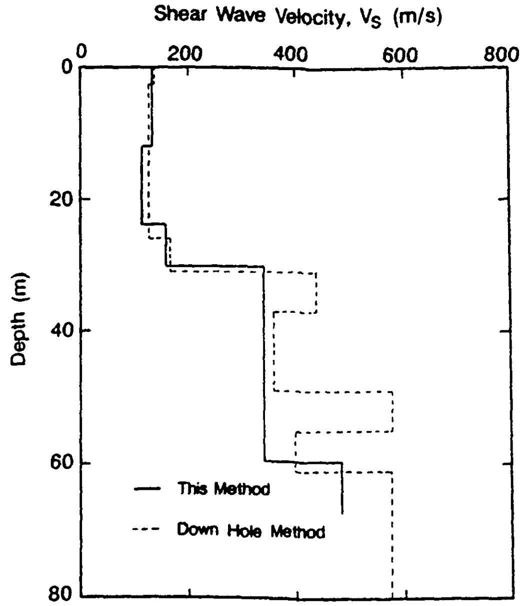

Fig. 10 V_s profiles obtained by different methods at Daiba

be achieved only if the effects of higher modes on Rayleigh wave propagation are taken into account.

Fig. 10 shows the V_s profile estimated from the inversion. Also shown in Fig. 10 is the V_s profile from Fig. 7. The estimated shear wave velocities for the surface layers of silty clay and clay are 150 m/s, and for the layers below 30 m depth are more than 300 m/s. The estimated shear wave velocities and the depth to the layer with V_s greater than 300 m/s are in fairly good agreement with those determined by the conventional method. The fairly good agreement in both V_s profile and depth to the stiff layer indicates that the Rayleigh wave method using short-period microtremors is promising for estimating V_s structures.

CONCLUSIONS

The use of short-period microtremors has been examined for the determination of V_s structures for estimating dynamic characteristics of a site. Microtremor measurements were made at two sites using an expanding circular array. Based on the high-resolution frequency-wavenumber spectrum analysis and the spacial auto-correlation analysis of the measured data, Rayleigh wave dispersion characteristics can be determined. The inverse process considering the effects of higher Rayleigh modes enables one to estimate the V_s structure of the site. The comparison of these results with the available data leads to the following conclusions:

(1) The dispersion curve determined from microtremor measurements is consistent with that from the steady-state Rayleigh wave method.

(2) The F-K spectrum analysis and the auto-correlation analysis of microtremor data can yield almost the same dispersion curves.

(3) The V_s profiles inferred from the inverse analysis using the observed dispersion curves are in good accord with those obtained by the down-hole method.

It should be recognized that the Rayleigh wave method using microtremors may yield V_s structures which are less accurate than does the conventional down-hole method. Nevertheless, the above results and findings suggest that the use of short-period microtremors is promising as an economical mean of characterizing site conditions.

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